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Improving Online Railway Deadlock Detection using a Partial Order Reduction

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Acknowledgments

Project context: **GoTo – Greater Oslo Area Train Optimization**, with Norwegian railway infrastructure manager Bane NOR.

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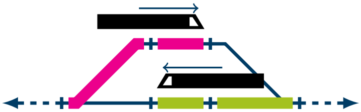
- Carlo Mannino, SINTEF Digital, Oslo

Automation and autonomy in railway

- Tight schedules are often disrupted by unforseen events.
- Manual dispatching: operators try to re-schedule.
- Autonomous dispatching: automatically compute optimal schedules.
  - ✓ Sensors are connected to online computer system.
  - ✓ Optimization tools can compute good or optimal schedules in real time.
  - ✗ ... for large infrastructures
  - ✗ ... with direct control of trains

Schedules and deadlocks



- Even finding a feasible schedule in a real-time dispatching situation is NP-hard[1].
- Situations change in minutes.
- Large scale re-scheduling systems are likely to use heuristics and/or limited scheduling horizons.
- no feasible schedule $\Leftrightarrow$ bound for deadlock
- Check for deadlocks as a separate procedure?
- Found to be hard in a 2021 study[2].

[2] Lu, Dessouky, and Leachman, “Modeling train movements through complex rail networks”.
[2] Sasso et al., “The Tick Formulation for deadlock detection and avoidance in railways traffic control”.

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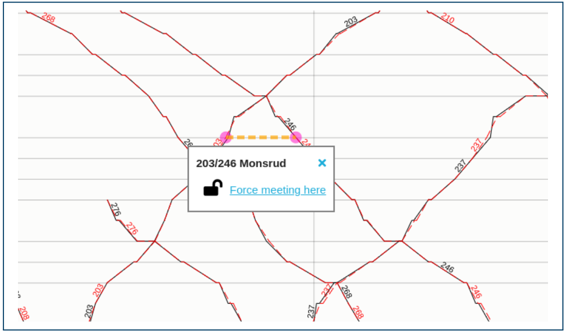
Manual overrides



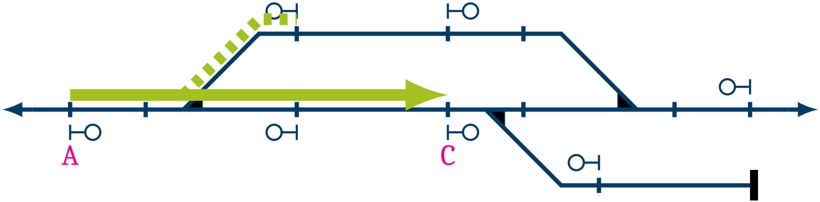
- Even with a working autonomous re-scheduling and dispatching, operators might want to override decisions.
- This may cause deadlocks.

Deadlocks in manual dispatching

Photo by Nate Beal (CC-BY-2.0)

- With manual dispatching of long trains, actual deadlocks happen in practice.
- These require costly recovery operations.
- Beneficial to know about deadlocks as early as possible.

Train movements



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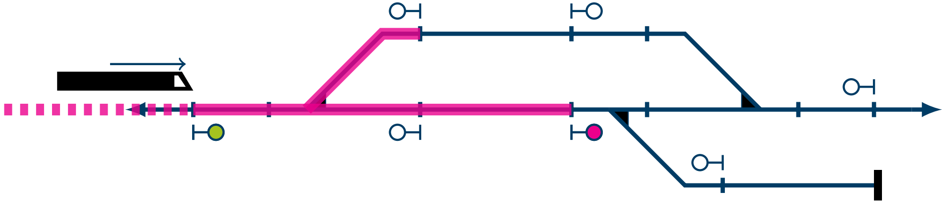
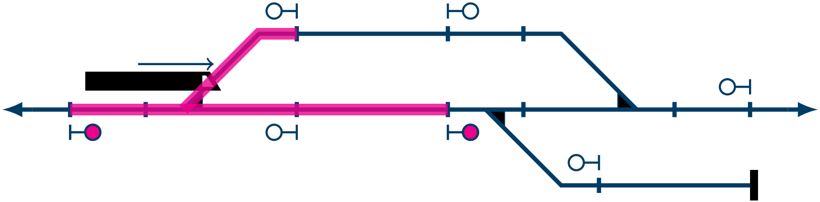
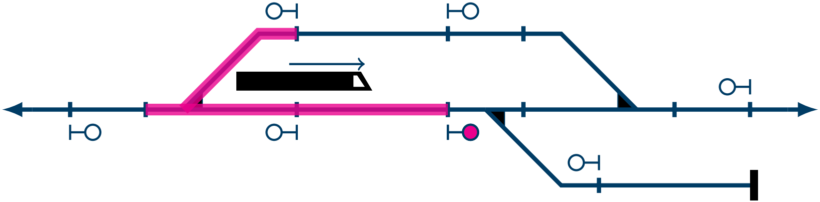
Train movements



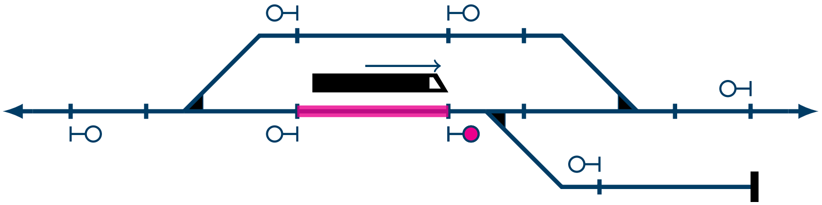
Train movements



Train movements



Train movements



Approach to the deadlock detection problem

The route system already supplies a discretization for us!

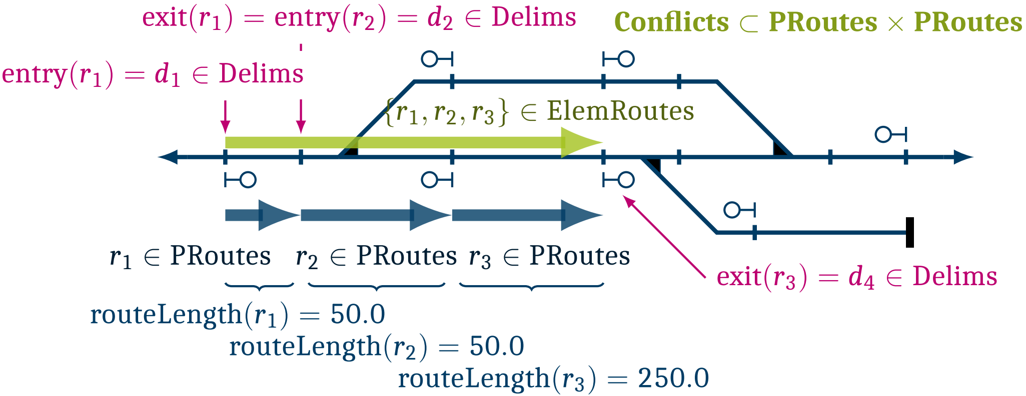
For deadlock detection, we can ignore:

- velocities and exact locations
- acceleration/braking power
- sight distances

Solution idea:

1. Define problem using route infrastructure model.
2. Model as a (discrete) transition system.
3. Solve with a SAT solver.

Infrastructure model I



Trains model T

In addition to infrastructure, we need the following train data:

- A set of trains, Trains.
- The length of each train, $\text{trainLength} : \text{Trains} \to \mathbb{R}$.
- Each train's initial position, $\text{initialRoutes} : \text{Trains} \to 2^{\text{PRoutes}}$.
- Each train's final position alternatives, $\text{finalRoutes} : \text{Trains} \to 2^{\text{PRoutes}}$.

Online railway deadlock detection problem

Definition

- The online railway deadlock detection problem $D = (I, T)$ ...
- ... is solved by a deadlock detection algorithm $d : D \rightarrow \{\text{Live}, \text{Dead}\}$, which returns Live if all trains can travel to one of their final positions, and Dead otherwise.

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Transition system model[3]

- Propositional logic: k-step unrolling of transition relation:

$$\Phi_k = \bigwedge_{i=0}^{k} \phi_i$$

- Variables for state i:
  - For each partial route r: One-hot encoding of

$$o_r^i \in \text{Trains} \cup \{\text{Free}\}$$

  - Does the train reach its destination in or before state i?

$$f_t^i$$

[3] Based on Luteberget et al., "SAT modulo discrete event simulation applied to railway design capacity analysis".

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Specifying the transition relation

- First attempts at planning with SAT used classical frame axioms[4]: one action in each step and consecutive states equal except for action effects.

$$\text{atMostOne}(\{\alpha\}), \qquad \alpha \Rightarrow (v^{i-1} \Rightarrow v^i), \qquad \text{for all } v \text{ not in effects}(\alpha)$$

- Usually, better encodings from explanatory frame axioms[5]: a changed value must be explained as an action effect. Thinking backwards!

$$(\neg v^{i-1} \wedge v^i) \Rightarrow \bigvee \alpha, \qquad \text{(any } \alpha \text{ with effect v)}$$

- Non-interfering actions can happen in the same step!

[4] Kautz and Selman, "Planning as Satisfiability".
[5] Kautz, McAllester, and Selman, "Encoding Plans in Propositional Logic".

Constraints (1)

- Mutual exclusion between conflicting routes:

$$\bigwedge_{(a,b)\in \text{Conflicts}} \left((o_a^i = \text{Free}) \vee (o_b^i = \text{Free}) \right)$$

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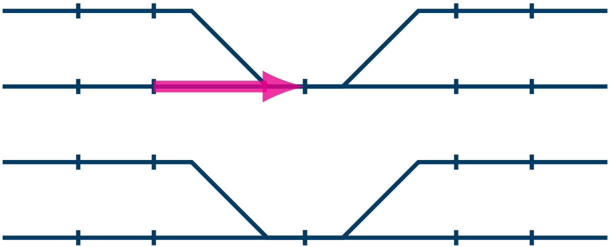
Constraints (2)

Path consistency:

$$\left(o_r^{i-1} \neq t \wedge o_r^{i} = t\right) \Rightarrow \ldots$$

State $i-1$

State i



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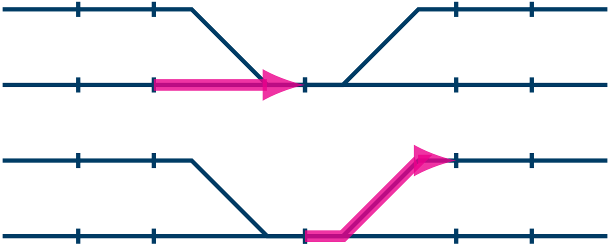
Constraints (2)

Path consistency:

$$\left(o_r^{i-1} \neq t \wedge o_r^{i} = t\right) \Rightarrow \bigvee_{\substack{x \in \text{PRoutes} \\ \text{entry}(r)=\text{exit}(x)}} o_x^{i-1} = t$$

State $i-1$

State i



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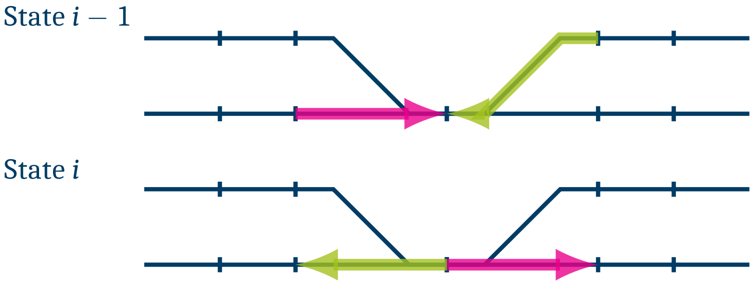
Constraints (2)

Path consistency:

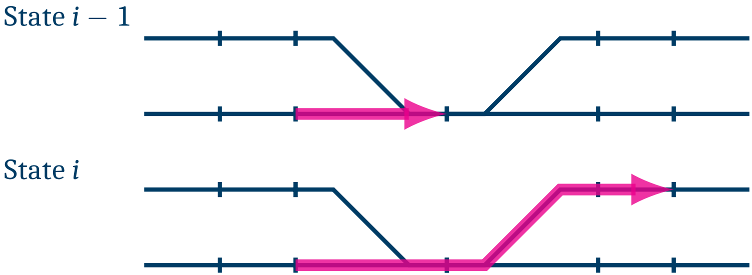
$$\left(o_r^{i-1} \neq t \wedge o_r^{i} = t\right) \Rightarrow \bigvee_{\substack{x \in \text{PRoutes} \\ \text{entry}(r) = \text{exit}(x)}} \left(o_x^{i-1} = t \wedge o_x^{i} = t\right)$$



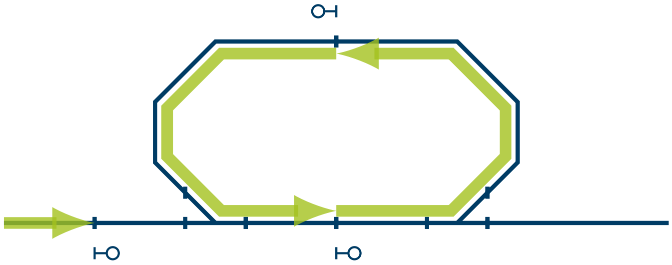
Constraints (2)

Path consistency:

$$\left(o_r^{i-1} \neq t \wedge o_r^i = t\right) \Rightarrow \bigvee_{\substack{x \in \text{PRoutes} \\ \text{entry}(r) = \text{exit}(x)}} o_x^i = t$$



Path consistency in cyclic infrastructure



- Can use cycle elimination constraints[6].
- This a general issue with logical encoding of problems involving graphs[7].

[6] Luteberget et al., "SAT modulo discrete event simulation applied to railway design capacity analysis".
[7] Gebser, Janhunen, and Rintanen, "SAT Modulo Graphs: Acyclicity".

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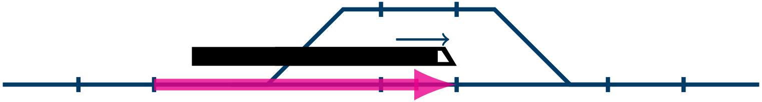
Constraints (3)

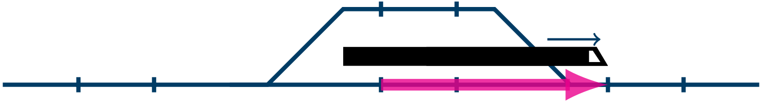
Train persistence:

$$\left(o_r^{i-1} = t \land o_r^i \neq t\right) \Rightarrow \ldots$$

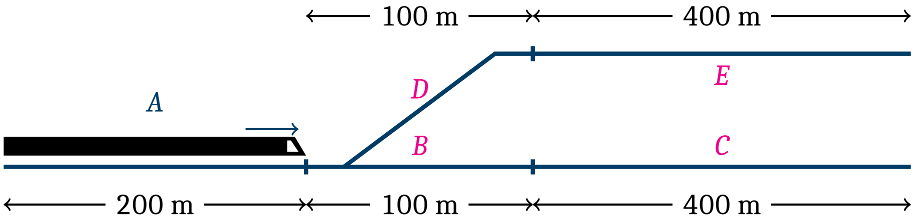
State $i-1$



State i



Freeing combinations



If A holds a train t of length 200.0 m, freeing A is constrained by:

$$A^{i-1} \Rightarrow \left(A^i \vee (B^i \wedge C^i) \vee (D^i \wedge E^i)\right).$$

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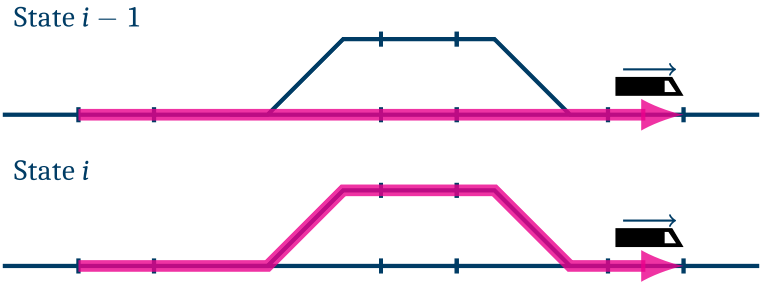
Constraints (3)

Train persistence:

$$\left(o_r^{i-1} = t \wedge o_r^i \neq t\right) \Rightarrow \text{freeable}_t^i(r, \text{trainLength}(t))$$



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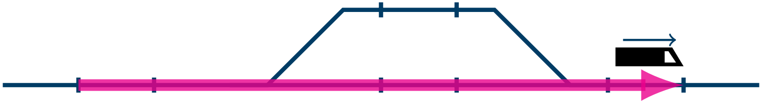
Constraints (3)

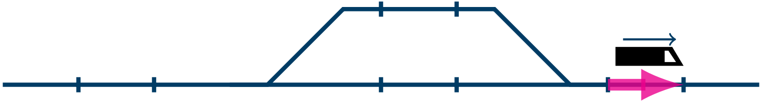
Train persistence:

$$(o_r^{i-1} = t) \Rightarrow (o_r^i \neq t \Leftrightarrow \text{freeable}_t^i(r, \text{trainLength}(t)))$$

State $i - 1$



State i



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The transition system

Putting the constraints together, ϕ_i is the conjunction of:

- **Mutual exclusion:** $\bigwedge_{(a,b)\in\text{Conflicts}} \left((o_a^i = \text{Free}) \vee (o_b^i = \text{Free})\right)$
- **Path consistency:** $\left(o_r^{i-1} \neq t \wedge o_r^i = t\right) \Rightarrow \bigvee_{\substack{x\in\text{PRoutes} \\ \text{entry}(r)=\text{exit}(x)}} o_x^i = t$
- **Train persistence:** $(o_r^{i-1} = t) \Rightarrow (o_r^i \neq t \Leftrightarrow \text{freeable}_t^i(r, \text{trainLength}(t)))$
- **Elementary routes:** $\bigwedge_{e\in\text{ElemRoutes}} \bigwedge_{r\in e} \left(\left(o_r^{i-1} \neq t \wedge o_r^i = t\right) \Rightarrow \bigwedge_{r\in e}(o_r^i = t)\right)$

We also have a known **initial state** ϕ_0, and a **goal condition** $G_i = \bigwedge_{t\in\text{Trains}} f_t^i$, where:

$$(\neg f_t^{i-1} \wedge f_t^i) \Rightarrow \bigvee_{r\in\text{finalRoutes}(t)} o_r^i = t$$

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Complete bounded model checking

- Bounded model checking[8] is not complete unless the number of transitions exceeds the completeness threshold.
- Completeness threshold for acyclic route-based railway model[9]: longest possible path, summed over trains.

Algorithm 1: Deadlock detection using incremental k-bounded model checking

---

Input : A problem instance $D = (I, T)$ and a bound k.

Output: Dead if the system is bound for deadlock, Live otherwise.

1 **let** $i = 1$.

2 **if** $\Phi_i \land G_i$ is Sat, **return** Live

3 **if** $i < k$, increment i and go to 2, **else return** Dead

---

[8] Biere et al., "Bounded model checking".

[9] Sasso et al., "The Tick Formulation for deadlock detection and avoidance in railways traffic control".

Instance				Ticks MIP alg. (reported in [21])		Algorithm 1	
#	Result	n_r	n_t	Steps	Time (s)	Steps	Time (s)
01	LIVE	14	3	8	1.08	5	0.00
02	DEAD	14	3	8	0.98	10	0.00
03	LIVE	14	3	8	0.93	5	0.00
04	LIVE	30	2	15	1.20	4	0.00
05	LIVE	30	3	20	1.31	5	0.00
06	DEAD	30	3	20	2.78	19	0.03
07	DEAD	38	5	34	37.31	34	0.17
08	LIVE	46	5	33	1.78	5	0.00
09	DEAD	38	6	37	>60.00	37	0.26
10	DEAD	38	7	42	4.23	42	0.02
11	DEAD	62	2	27	17.60	26	3.30
12	DEAD	62	4	39	>60.00	40	1.70
13	DEAD	62	4	39	>60.00	40	1.30
14	LIVE	62	4	39	3.27	6	0.00
15	DEAD	46	4	42	>60.00	42	0.22
16	LIVE	62	5	50	5.33	5	0.00
17	LIVE	62	4	50	43.11	6	0.00
18	DEAD	62	4	50	>60.00	49	2.10
19	DEAD	62	5	51	>60.00	50	0.83
20	DEAD	70	5	57	>60.00	56	1.20

Zig-zag algorithm[10]

Idea: at least one route must be allocated in each transition:

$$z_i = \bigvee_{t \in \text{Trains}} \bigvee_{r \in \text{PRoutes}} ((o_r^{i-1} \neq t) \wedge (o_r^i = t)), \qquad Z_i = \bigwedge_{j=1}^{i} z_j$$

Algorithm 2: Online railway deadlock detection with global progress constraint

---

Input : A problem instance $D = (I, T)$.

Output: Dead if the system is bound for deadlock, Live otherwise.

1 **let** $i = 1$.

2 **if** $\Phi_i \wedge Z_i$ is Unsat, **return** Dead

3 **if** $\Phi_i \wedge Z_i \wedge G_i$ is Sat, **return** Live

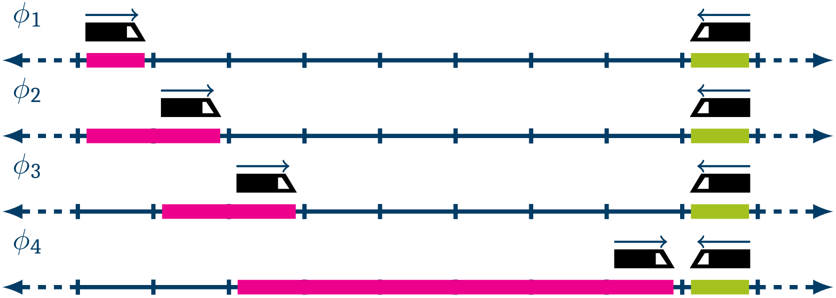
4 increment i and go to 2.

---

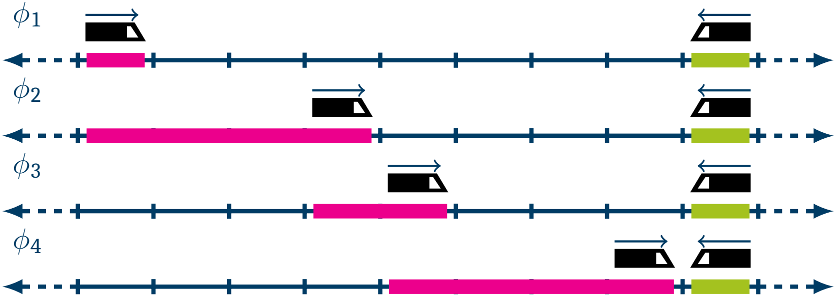
[10] Eén and Sörensson, "Temporal induction by incremental SAT solving".

Instance				Ticks MIP alg. (reported in [21])		Algorithm 1		Algorithm 2	
#	Result	n_r	n_t	Steps	Time (s)	Steps	Time (s)	Steps	Time (s)
01	LIVE	14	3	8	1.08	5	0.00	5	0.00
02	DEAD	14	3	8	0.98	10	0.00	7	0.00
03	LIVE	14	3	8	0.93	5	0.00	5	0.00
04	LIVE	30	2	15	1.20	4	0.00	4	0.00
05	LIVE	30	3	20	1.31	5	0.00	5	0.00
06	DEAD	30	3	20	2.78	19	0.03	9	0.04
07	DEAD	38	5	34	37.31	34	0.17	7	0.00
08	LIVE	46	5	33	1.78	5	0.00	5	0.00
09	DEAD	38	6	37	>60.00	37	0.26	14	0.25
10	DEAD	38	7	42	4.23	42	0.02	2	0.00
11	DEAD	62	2	27	17.60	26	3.30	15	3.30
12	DEAD	62	4	39	>60.00	40	1.70	20	9.60
13	DEAD	62	4	39	>60.00	40	1.30	20	11.00
14	LIVE	62	4	39	3.27	6	0.00	6	0.01
15	DEAD	46	4	42	>60.00	42	0.22	15	1.30
16	LIVE	62	5	50	5.33	5	0.00	5	0.00
17	LIVE	62	4	50	43.11	6	0.00	6	0.01
18	DEAD	62	4	50	>60.00	49	2.10	15	13.10
19	DEAD	62	5	51	>60.00	50	0.83	16	36.00
20	DEAD	70	5	57	>60.00	56	1.20	-	>60.00

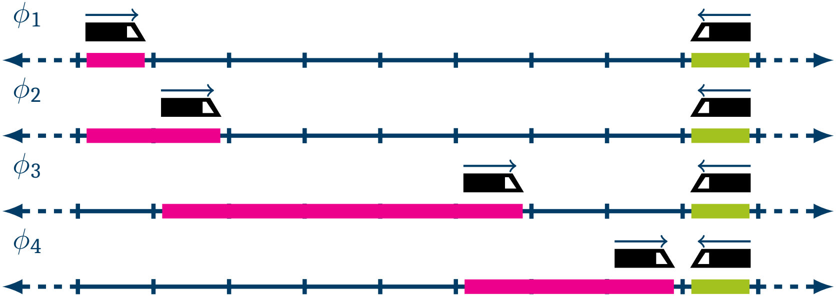
Symmetries



Symmetries



Symmetries



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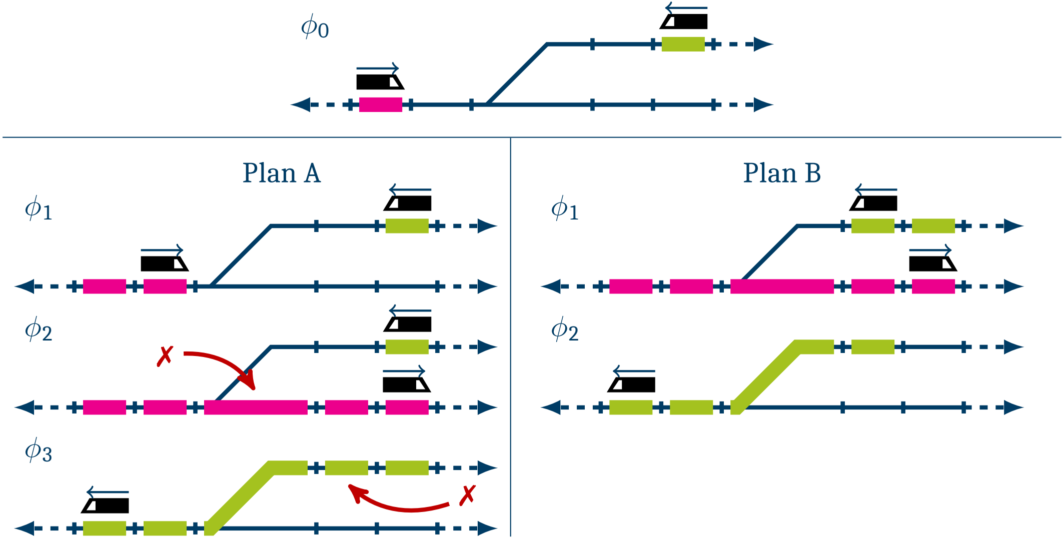
Maximal progress

- For any allocation that is extending the path from the previous state,
- ... a route conflicting with r must be occupied in the previous state.
- Same idea as process semantics for planning as SAT[11]: all actions happen as early as possible.

$$\left(o_r^{i-1} \neq t \wedge o_r^{i} = t \wedge \bigvee_{\substack{x \in \text{PRoutes} \\ \text{entry}(r)=\text{exit}(x)}} (o_x^{i-1} = t) \right) \Rightarrow \bigvee_{\substack{(r,y) \in \text{Conflicts} \\ \text{...or } y=r}} (o_y^{i-1} \neq t \wedge o_y^{i-1} \neq \text{Free})$$

[11] Rintanen, Heljanko, and Niemelä, "Planning as satisfiability: parallel plans and algorithms for plan search".



ϕ_0
Plan A
ϕ_1
ϕ_2
ϕ_3
Plan B
ϕ_1
ϕ_2

Maximal progress

We call this constraint a partial order reduction because it creates a unique representation of all solutions that represent the same partial order containing the trains' paths and the order in which they use conflicting routes.

Algorithm 3: Online railway deadlock detection with partial order reduction

---

Input : A problem instance $D = (I, T)$.
Output: Dead if the system is bound for deadlock, Live otherwise.

1 **let** $i = 1$.
2 **if** $\Phi_i \land Z_i \land P_i$ is Unsat, **return** Dead
3 **if** $\Phi_i \land Z_i \land P_i \land G_i$ is Sat, **return** Live
4 increment i and go to 2.

Instance				Ticks MIP alg. (reported in [21])		Algorithm 1		Algorithm 2		Algorithm 3	
#	Result	n_r	n_t	Steps	Time (s)	Steps	Time (s)	Steps	Time (s)	Steps	Time (s)
01	LIVE	14	3	8	1.08	5	0.00	5	0.00	5	0.00
02	DEAD	14	3	8	0.98	10	0.00	7	0.00	5	0.00
03	LIVE	14	3	8	0.93	5	0.00	5	0.00	5	0.00
04	LIVE	30	2	15	1.20	4	0.00	4	0.00	4	0.00
05	LIVE	30	3	20	1.31	5	0.00	5	0.00	5	0.00
06	DEAD	30	3	20	2.78	19	0.03	9	0.04	5	0.00
07	DEAD	38	5	34	37.31	34	0.17	7	0.00	5	0.00
08	LIVE	46	5	33	1.78	5	0.00	5	0.00	5	0.00
09	DEAD	38	6	37	>60.00	37	0.26	14	0.25	7	0.01
10	DEAD	38	7	42	4.23	42	0.02	2	0.00	2	0.00
11	DEAD	62	2	27	17.60	26	3.30	15	3.30	3	0.00
12	DEAD	62	4	39	>60.00	40	1.70	20	9.60	8	0.19
13	DEAD	62	4	39	>60.00	40	1.30	20	11.00	8	0.12
14	LIVE	62	4	39	3.27	6	0.00	6	0.01	6	0.02
15	DEAD	46	4	42	>60.00	42	0.22	15	1.30	6	0.01
16	LIVE	62	5	50	5.33	5	0.00	5	0.00	5	0.01
17	LIVE	62	4	50	43.11	6	0.00	6	0.01	6	0.02
18	DEAD	62	4	50	>60.00	49	2.10	15	13.10	6	0.05
19	DEAD	62	5	51	>60.00	50	0.83	16	36.00	6	0.03
20	DEAD	70	5	57	>60.00	56	1.20	-	>60.00	6	0.03

Conclusion

Adapted a transition system model for railway planning from[12] to the online railway deadlock detection problem.

- Shows improved performance over earlier work[13] because of:
  - Specifying the transition relation with more parallelism, decreasing the minimum number of transitions to find a feasible schedule.
  - Partial order reduction, decreasing the maximum number of transitions to find a bound-for-deadlock situation.
  - Using a SAT solver (instead of MIP)

Future work:

- Abstraction refinement to find deadlocks in larger networks
- Integration with a scheduling algorithm for producing deadlock-free schedules

[12] Luteberget et al., "SAT modulo discrete event simulation applied to railway design capacity analysis".
[13] Sasso et al., "The Tick Formulation for deadlock detection and avoidance in railways traffic control".